Optimal Approximation Rates for Deep ReLU Neural Networks on Sobolev Spaces

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Deep ReLU neural networks are among the most widely used class of neural networks in practical applications. We consider the problem of determining optimal L_p-approximation rates for deep ReLU neural networks on the Sobolev class $W^s(L_q)$ for all $1 \leq p, q \leq \infty$ and $s > 0$. Existing sharp results are only available when $q = \infty$, i.e. when the derivatives are measured in L_∞. In our work, we extend these results and determine the best possible rates for all p, q and s for which a compact Sobolev embedding holds, i.e. when $s/d > 1/q - 1/p$. This settles in particular the classical non-linear regime where $p > q$. Our techniques can also be used to obtain optimal rates for Besov spaces. We will discuss some of the technical details of the proof and conclude by giving a few open research directions.