Approximation of smooth functions by short exponential sums

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We consider function approximation using exponential sums of the form

$$f(t) = \sum_{j=1}^{M} \gamma_j \, \mathrm{e}^{\phi_j t} = \sum_{j=1}^{M} \gamma_j \, z_j^t,$$

where $M \in \mathbb{N}$, $\gamma_j \in \mathbb{C} \setminus \{0\}$, and $z_j = \mathrm{e}^{\phi_j} \in \mathbb{C} \setminus \{0\}$ with $\phi_j \in \mathbb{C}$ are pairwise distinct.

The exponential sum model can be well applied to periodic as well non-periodic function approximation. However, exponential decay of approximation errors in finite intervals and on $[0, \infty)$ has been shown only for special completely monotonic functions as $f(t) = (1+t)^{-1}$, while numerical experiments show very good approximation results also for other functions as Bessel functions or the Gaussian function.

There are several algorithms around to obtain exponential sum approximations. Beside the very expensive generalized Remez algorithm there exist different suboptimal algorithms based on Prony's method, which employ function values, derivative values or moments of the function to be approximated. Further, there is a close connection between rational approximation in frequency domain and the approximation by exponential sums in spatial domain.

In the talk, we give a survey on recents results on function approximation by exponential sums and corresponding numerical algorithms.

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