On sharp L^2 convergence rates for kernel based interpolation

Tizian Wenzel
University of Stuttgart, Germany
tizian.wenzel@mathematik.uni-stuttgart.de

We consider the interpolation of functions in reproducing kernel Hilbert spaces (RKHS) based on function values. It is well known that the distribution of the interpolation points has a crucial influence on the accuracy of the interpolant.
First, we deal with direct estimates, where we recall standard L^2 convergence rates and show improved L^∞ convergence rates for adaptively selected interpolation points. This convergence analysis relies on the analysis of the pointwise worst case error over all functions from the RKHS.
Second, we deal with inverse estimates and show that a large enough L^2 convergence rate implies that a function is contained in the RKHS or intermediate Sobolev spaces. In contrast to the previous analysis, we consider here the worst case error over any set of well distributed interpolation points.
Altogether this shows a one-to-one correspondence between smoothness and approximation rate of a function.
The results are illustrated by numerical examples.

Joint work with Gabriele Santin (Fondazione Bruno Kessler, Trento, Italy), Bernard Haasdonk (University of Stuttgart, Stuttgart, Germany).