A varifold perspective on discrete surfaces

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We propose a natural framework for the study of surfaces and their different discretizations based on varifolds. Varifolds have been introduced by Almgren to carry out the study of minimal surfaces. Though mainly used in the context of rectifiable sets, they turn out to be well suited to the study of discrete type objects as well.

While the structure of varifold is flexible enough to adapt to both regular and discrete objects, it allows to define variational notions of mean curvature and second fundamental form based on the divergence theorem. Thanks to a regularization of these weak formulations, we propose a notion of discrete curvature (actually a family of discrete curvatures associated with a regularization scale) relying only on the varifold structure. We prove nice convergence properties involving a natural growth assumption: the scale of regularization must be large with respect to the accuracy of the discretization.

However, such accuracy of the discretization involves the so called bounded Lipschitz distance between the varifold associated with the discrete object and the underlying regular varifold, while usual data are not directly provided with a varifold structure (we think of point cloud data without weights nor tangential directions). We hence raise the issue of infering the varifold structure from data points. More precisely, we assume that a continuous object S is given through a probability measure μ supported in S and our data are obtained by sampling μ with N points: $(X_1, \ldots, X_N) \sim \mu$ is an i.i.d. sample and our data is an instance of the empirical measure

$$\mu_N = \frac{1}{N} \sum_{i=1}^{N} \delta_{X_i} \, .$$

We then investigate the definition and convergence of an estimator of the varifold associated with S.

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