Metric Approximation of Set-Valued Functions

Nira Dyn
School of Mathematical Sciences, Tel Aviv University, Israel
niradyn@tauex.tau.ac.il

Abstract : This talk is a review of ideas in the approximation of Set-Valued functions, mapping a closed interval to the space of all non-empty, compact subsets of $\mathbb{R}^d$. These ideas have evolved in a series of papers, written jointly, first with Elza Farkhi and Alona Mokhov, and later also with Elena Berdysheva. We were inspired by the first paper on the approximation of these set-valued functions, authored by Z. Artstein in the late eighties. Most papars on that subject have studied the class of set-valued functions with values restricted to convex sets, based on operations such as Minkowski sum of sets and the Auman integral. The new tool in Artstein's , paper is the "'metric average", with which a "'piecewise linear" interpolant is constructed. In our papers we introduced several more metric tools, such as metric chains, metric divided differences, weighted metric integral, with which we adapt to set-valued functions classical, sampled-based, linear approximation operators, Fourier partial sums, and several positive integral operators. The metric adaptation yields approximants with similar approximation ordersas in the real-valued case, for continuous SVFs, and for SVFs of bounded variation.

Joint work with Elena Berdysheva, (Cape-Town University, South-Africa), Elza Farkhi, (Tel Aviv University, Israel) and Alona Mokhov, (Afeka, Tel Aviv College of Engineering, Israel).