Solving the shallow-water semi-geostrophic equations through Sinkhorn's algorithm

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We propose a novel application of the generic Sinkhorn algorithm as proposed by L. Chizat et al. (2016), to solve the shallow-water semi-geostrophic (SWSG) equations. The method provides a numerical and geometrically intuitive algorithm to model large-scale oceanic phenomena through the methods devised by Cullen and Gangbo (2001), which are based on the Cullen-Purser Stability Principle (Cullen and Purser, 1984). Specifically, we solve the SWSG equations stated as an optimal transport problem, with one L^2-norm divergence and one indicator function divergence imposed on the marginals. Solutions of the equations correspond to energy minimisation states which allow us to cast the problem as an OT one. We are then able to artificially add KL regularisation so that we can leverage the results of Chizat et al, in particular the Sinkhorn updates for generic marginal constraints.

In this first phase we examine the initialisation process. Initialisation is achieved by starting from an analytical ocean height, which can be mapped to the geostrophic domain through Hoskin's transformations. The algorithm is then given these known geostrophic densities and calculates the corresponding minimum height associated to this density. In this case, height solutions correspond to the analytical height which we input. We check convergence for these known initial states and investigate best practises for keeping errors low.

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