Minimax estimation of discontinuous optimal transport maps: The semi-discrete case

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We consider the problem of estimating the optimal transport map between two probability distributions, P and Q in $\mathbb{R}^d$, on the basis of i.i.d. samples. All existing statistical analyses of this problem require the assumption that the transport map is Lipschitz, a strong requirement that, in particular, excludes any examples where the transport map is discontinuous. As a first step towards developing estimation procedures for discontinuous maps, we consider the important special case where the data distribution Q is a discrete measure supported on a finite number of points in $\mathbb{R}^d$. We study a computationally efficient estimator initially proposed by [PNW21], based on entropic optimal transport, and show in the semi-discrete setting that it converges at the minimax-optimal rate $n^{-1/2}$, independent of dimension. Other standard map estimation techniques both lack finite-sample guarantees in this setting and provably suffer from the curse of dimensionality. We confirm these results in numerical experiments, and provide experiments for other settings, not covered by our theory, which indicate that the entropic estimator is a promising methodology for other discontinuous transport map estimation problems.

[PNW21] Pooladian, Aram-Alexandre, and Niles-Weed, Jonathan. "Entropic estimation of optimal transport maps", preprint, 2021

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