Numerical solution of the isotropic Landau–Lifshitz equation using the Hasimoto transform

Georg Maierhofer
Sorbonne Université, France
georg.maierhofer@sorbonne-universite.fr

The isotropic Landau–Lifshitz (LL) equation provides a model for a wide range of physical phenomena describing inter alia magnetization dynamics in ferromagnetism and the evolution of vortex filaments in ideal fluids. The fully nonlinear structure of this equation makes computations exceedingly difficult and prior approaches had to resort to comparatively expensive implicit formulations to achieve stable approximations to this equation.

In this work, we introduce a novel numerical approach to computing solutions to the LL equation based on the Hasimoto transform which relates the LL flow to a cubic nonlinear Schrödinger (NLS) equation. In exploiting this nonlinear transform we are able to introduce the first fully explicit unconditionally stable symmetric integrators for the LL equation. Our approach consists of two parts: an integration of the NLS equation followed by the numerical evaluation of the Hasimoto transform. Motivated by the desire to study rough solutions to the LL equation, we also introduce a new symmetric low-regularity integrator for the NLS equation. This is combined with our novel fast low-regularity Hasimoto (FLowRH) transform, based on a tailored analysis of the resonance structures in the Magnus expansion and a fast realisation based on block-Toeplitz partitions, to yield an efficient low-regularity integrator for the LL equation.

In this poster we will present an overview of the methodology of this novel approach whose favorable properties are exhibited both in theoretical convergence analysis and in numerical experiments.

Joint work with Valeria Banica (Sorbonne Université, France) and Katharina Schratz (Sorbonne Université, France).