The periodic KdV equation: Computing with nonlinear Fourier series

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The integrability of the Korteweg-de Vries (KdV) equation affords an inverse scattering transform (IST) that "solves" the associated initial-value problem. First, historically, this fact allowed one to determine a class of explicit solutions, including both soliton solutions and (almost) periodic so-called finite-genus solutions. Then the IST allowed one to compute the long-time behavior of the solution to the Cauchy problem on the line using the Deift-Zhou method of nonlinear steepest descent. More recently, the IST has been used to compute solutions of the Cauchy problem on the line in the entire space-time plane.

In this talk, I will discuss a recent development that allows one to compute finite-genus solutions when the genus is large and to then effectively approximate the solution of the KdV equation with periodic initial data. This new approach uses a Riemann-Hilbert problem posed on (possibly thousands of) intervals and gives a natural interpretation as computing a nonlinear Fourier series approximation of the solution under consideration. Implications for dispersive quantization will be discussed.

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