Adaptive Approximations for PDE-Constrained Parabolic Control Problems

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Numerical solvers for PDEs have matured over the past decades in efficiency, largely due to the development of sophisticated algorithms based on closely intertwining theory with numerical analysis. Consequently, systems of PDEs as they arise from optimization problems with PDE constraints also have become more and more tractable.

Optimization problems constrained by a parabolic evolution PDE are challenging from a computational point of view, as they require to solve a system of PDEs coupled globally in time and space. For their solution, time-stepping methods quickly reach their limitations due to the enormous demand for storage. An alternative approach is a full space-time weak formulation of the parabolic PDE which allows one to treat the constraining PDE as an operator equation without distinction of the time and space variables. An optimization problem constrained by a parabolic PDE in full space-time weak form leads to a coupled system of corresponding operator equations which is, of course, still coupled globally in space and time.

For the numerical solution of such coupled PDE systems, adaptive methods appear to be most promising, as they aim at distributing the available degrees of freedom in an a-posteriori-fashion to capture singularities in the data or domain. Employing wavelet schemes, we can prove convergence and optimal complexity. I will also address results on corresponding finite element approximations.

The theoretical basis for proving convergence and optimality of wavelet-based algorithms for such type of coupled PDEs is nonlinear approximation theory and the characterization of solutions of PDEs in Besov spaces.

Finally, I would like to address the possibility of solving such coupled PDEs by neural networks, combined with the characterization of solutions of PDEs in Barron spaces.