Laplacian Eigenfunctions that Do Not Feel Boundary: Theory, Computation, and Applications

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I will discuss Laplacian eigenfunctions defined on a Euclidean domain of general shape, which "do not feel the boundary." These Laplacian eigenfunctions satisfy the Helmholtz equation inside the domain and can be extended smoothly and harmonically outside of the domain. These eigenfunctions satisfy the "nonlocal" boundary conditions, not the usual Dirichlet or Neumann boundary conditions. However, they can be computed via the eigenanalysis of the integral operator with rather simple potential kernel. The key here is the use of integral kernel that is a function of "distance" between a pair of points in a given domain. Compared to directly solving the Helmholtz equations on such domains, the eigenanalysis of this integral operator has several advantages including the numerical stability and amenability to modern fast numerical algorithms (e.g., the Fast Multipole Method). I will also discuss its relationship with the Laplacians satisfying various standard boundary conditions (e.g., Dirichlet, Neumann, Robin, periodic, anti-periodic, etc.) as well as the Krein-von Neumann Laplacian. Finally, I will discuss the extension of this integral operator approach to combinatorial graphs and simplicial complexes as its domain, and certain applications, e.g., image classification, heat equations, and characterization of biological shapes.