Trefftz and quasi-Trefftz methods

Lise-Marie Imbert-Gérard
University of Arizona, USA
lmig@math.arizona.edu

In the field of numerical partial differential equations (PDEs), many methods rely on polynomial bases. These are very versatile tools, for instance in terms of their approximating properties: it is well-known that high order properties can be achieved to approximate any function with high enough regularity by choosing a sufficiently large space of polynomials.

Alternatively, various methods rely on problem-dependent bases, hence incorporating knowledge from the governing PDE into the basis functions. Such bases can achieve high order properties to approximate any sufficiently regular function **satisfying the governing PDE**, at a lower cost in terms of degrees of freedom compared to polynomial bases. However they cannot do so to approximate any function with high enough regularity.

Several methods leveraging PDE-dependent functions have been developed under the name of Trefftz methods. This talk will focus on Discontinuous Galerkin types of Trefftz methods (TDG methods), in the particular context of time-harmonic wave propagation problems. Here information about the ambient medium is incorporated into oscillating basis functions via the appropriate wave-number. Some properties and drawbacks of basis of oscillating functions will be presented.

One limitation of TDG methods is that explicit basis functions that solve the PDE are needed. But wave propagation in inhomogeneous media is modeled by PDEs with variable coefficients, and in general no exact solutions are available in this case. Quasi-Trefftz methods have been introduced, in the case of the Helmholtz equation with variable coefficients, to address this problem: they do not rely on exact solutions to the PDE but instead on approximate solutions constructed locally. We will discuss the origin, the construction, and the properties of these so-called quasi-Trefftz functions. We will also discuss the consistency error introduced in the TDG method by this construction process.