Efficient Finite Difference WENO Scheme for Hyperbolic Systems with Non-Conservative Products

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Higher order finite difference Weighted Essentially Non-Oscillatory (WENO) schemes have been constructed for conservation laws. For multidimensional problems, they offer high order accuracy at a fraction of the cost of a finite volume WENO or DG scheme of comparable accuracy. This makes them quite attractive for several science and engineering applications. But, to the best of our knowledge, such schemes have not been extended to non-linear hyperbolic systems with non-conservative products. In this paper, we perform such an extension which improves the domain of applicability of such schemes. The extension is carried out by writing the scheme in fluctuation form. We use the HLLI Riemann solver of Dumbser and Balsara (2016) as a building block for carrying out this extension. Because of the use of an HLL building block, the resulting scheme has a proper supersonic limit. The use of anti-diffusive fluxes ensures that stationary discontinuities can be preserved by the scheme, thus expanding its domain of applicability. Our new finite difference WENO formulation uses the same WENO reconstruction that was used in classical versions, making it very easy for users to transition over to the present formulation.

For conservation laws, the new finite difference WENO is shown to perform as well as the classical version of finite difference WENO, with two major advantages:- 1) It can capture jumps in stationary linearly degenerate wave families exactly. 2) It only requires the reconstruction to be applied once. Several examples from hyperbolic PDE systems with non-conservative products are shown which indicate that the scheme works and achieves its design order of accuracy for smooth multidimensional flows. Stringent Riemann problems and several novel multidimensional problems that are drawn from compressible Baer-Nunziato multiphase flow, multiphase debris flow and two-layer shallow water equations are also shown to document the robustness of the method. For some test problems that require well-balancing we have even been able to apply the scheme without any modification and obtain good results. Many useful PDEs may have stiff relaxation source terms for which the finite difference formulation of WENO is shown to provide some genuine advantages.

Joint work with Dinshaw S. Balsara (University of Notre Dame, USA), Chi-Wang Shu (Brown University, USA) and Harish Kumar (Indian Institute of Technology, Delhi, India).