Least-Squares Neural Network (LSNN) Method for Hyperbolic Conservation Laws

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Solutions of nonlinear hyperbolic conservation laws (HCLs) are often discontinuous due to shock formation; moreover, locations of shocks are a priori unknown. This presents a great challenge for traditional numerical methods because most of them are based on continuous or discontinuous piecewise polynomials on fixed meshes.

As an alternative, by employing a new class of approximating functions, neural network (NN), recently we proposed the least-squares neural network (LSNN) method for solving HCLs. The LSNN method shows a great potential to sharply capture shock without oscillation or smearing; moreover, its degrees of freedom are much less than those of mesh-based methods. Nevertheless, current iterative solvers for the LSNN discretization are computationally intensive and complicated.

In this talk, I will present our recent work on the LSNN for solving linear and nonlinear scalar HCLs.

[1] Cai, Z., Chen, J., and Liu, M., Least-squares ReLU neural network (LSNN) method for linear advection-reaction equation, J. Comput. Phys., 443 (2021) 110514.

[2] Cai, Z., Chen, J., and Liu, M., Least-squares ReLU neural network (LSNN) method for scalar nonlinear hyperbolic conservation law, Appl. Numer. Math., 174 (2022), 163-176.

[3] Cai, Z., Chen, J., and Liu, M., LSNN method for scalar nonlinear HCLs: discrete divergence operator, arXiv:2110.10895v2 [math.NA].

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