Dissipative solutions of compressible fluid flows: What do we approximate by structure preserving numerical schemes?

Mária Lukáčová
University of Mainz, Germany
lukacova@uni-mainz.de

In this talk we present an overview of our recent results on generalized dissipative solutions of compressible fluid flows. We will concentrate on the inviscid flows, the Euler equations, and mention also the relevant results obtained for viscous compressible flows, governed by the compressible Navier-Stokes equations.

The existence of dissipative solutions has been shown by the convergence analysis of suitable, invariant-domain preserving finite volume schemes [1,2,3] . In the case that the strong solution to the above equations exists, the dissipative solutions coincide with the strong solution on its lifespan. In this case we can also apply the relative entropy to derive rigorous error estimates between numerical solutions and the exact strong solution [4].

Otherwise, we apply a newly developed concept of K-convergence and prove the strong convergence of the empirical means of numerical solutions to a dissipative solution [5]. The latter is the expected value of the dissipative measure-valued solutions and satisfies a weak formulation of the Euler equations modulo the Reynolds stress tensor. In the class of dissipative solutions there exists a solution that is obtained as a vanishing viscosity limit of the Navier-Stokes system [6]. We will draw a connection to the Kolmogorov hypothesis and illustrated theoretical results by a series of numerical simulations.

If time permits, we present also our recent results on the error analysis of the Monte Carlo finite volume method for the approximation of statistical solutions of the compressible Navier-Stokes equations.

The present research has been partially supported by TRR 146 Multiscale simulation methods for soft matter systems, TRR 165 Waves to Weather funded by the German Science Foundation and by the Gutenberg Research College.

References

[1] E. Feireisl, M. Lukáčová-Medvid'ová, H. Mizerová, B. She, Numerical analysis of compressible fluid flows, Springer, 2021.

[2] E. Feireisl, M. Lukáčová-Medvid'ová, H. Mizerová, Convergence of finite volume schemes for the Euler equations via dissipative-measure valued solutions, *Found. Comput. Math.* **20** (2020), 923–966.

[3] E. Feireisl, M. Lukáčová-Medvid'ová, H. Mizerová, B. She, Convergence of a finite volume scheme for the compressible Navier-Stokes system, *ESAIM: Math. Model. Num.* **53** (2019), 1957–1979.

[4] M. Lukáčová-Medvid'ová, B. She, Y. Yuan, Error estimate of the Godunov method for multidimensional compressible Euler equations, *J. Sci. Comput.* **91** (2022), 71.

[5] E. Feireisl, M. Lukáčová-Medvid'ová, B. She, Y. Wang, Computing oscillatory solutions of the Euler system via K-convergence, *Math. Math. Models Methods Appl. Sci.* **31** (2021), 537–576.

[6] E. Feireisl, M. Lukáčová-Medvid'ová, S. Schneider, B. She, Approximating viscosity solutions of the Euler system, *Math. Comp.* **91** (2022), 2129-2164.

Joint work with Eduard Feireisl (Academy of Sciences, Prague, Czech Republic), Hana Mizerová (Comenius University, Bratislava, Slovakia), Bangwei She (Capital Normal University, Beijing, China) and Yuhuan Yuan (University of Mainz, Germany).