Time-fractional operators: From theory to simulation

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In this talk, we discuss several aspects of time-fractional differential operators. The non-local nature of the operator is challenging from a theoretical and algorithmic point of view but allows to account for memory effects in the mathematical model. We consider a wide class of time-fractional PDE models, including gradient flow, viscoelasticity, random field generators and polymeric fluids. Of special interest is the equivalence of the time-fractional gradient flow model with an integer order system in a higher dimension. Exploiting this equivalence allows us to construct efficient algorithms for time fractional PDEs. Replacing the kernel by a finite sum of exponentials transforms the non-local PDE into a integer-order system for the modes. To balance the discretization error in time and space with the kernel approximation error, the number of modes has to grow logarithmically. However, we observe numerically that the number of modes is very moderate if the finite sum of exponentials is based on a rational approximation of the kernel function. In this talk we present theoretical existence results, algorithmic aspects and show several applications in which fractional operators play an important role. Numerical examples illustrating the large flexibility of the proposed techniques.

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