Wavelet Galerkin Method for an Electromagnetic Scattering Problem

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The Helmholtz equation is challenging to solve numerically due to the pollution effect, which often results in a huge ill-conditioned linear system. In this paper, we present a high order wavelet Galerkin method to numerically solve an electromagnetic scattering from a large cavity problem modeled by the 2D Helmholtz equation. The high approximation order and the sparse stable linear system offered by wavelets are useful in dealing with the pollution effect. By using the direct approach presented in our past work [B. Han and M. Michelle, Appl. Comp. Harmon. Anal., 53 (2021), 270-331], we present various optimized spline biorthogonal wavelets on a bounded interval. We provide a self-contained proof to show that the tensor product of such wavelets form a 2D Riesz wavelet in the appropriate Sobolev space. Compared to the coefficient matrix of a standard Galerkin method, when an iterative scheme is applied to the coefficient matrix of our wavelet Galerkin method, much fewer iterations are needed for the relative residuals to be within a tolerance level. Furthermore, for a fixed wavenumber, the number of required iterations is practically independent of the size of the wavelet coefficient matrix. In contrast, when an iterative scheme is applied to the coefficient matrix of a standard Galerkin method, the number of required iterations doubles as the mesh size for each axis is halved. The implementation can also be done conveniently thanks to the simple structure, the refinability property, and the analytic expression of our wavelet bases.

Joint work with Bin Han (University of Alberta, Canada).