Low-rank tensor product approximations for radiative transfer in plane-parallel geometry

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The radiative transfer equation (RTE) has been established as a fundamental tool for the description of energy transport, absorption and scattering in many relevant scientific and societal applications, and requires numerical approximations. Classical numerical schemes in radiative transfer, such as the PN−and SN− approximations, exploit the tensor product structure of the underlying phase space, but they are affected by the so-called curse of dimensionality, which describes the exponential scaling of computational complexity with the physical dimension. For significant models like the RTE, this matter is particularly limiting.

The aim of our work is to introduce a low-rank tensor product framework for the approximation of the RTE. In the context of plane-parallel geometry, in order to tackle the dimensionality issue, we propose to construct tensor product solutions with low rank. An appropriate variational formulation, the even-parity formulation [1], allows us to recast the hyperbolic radiative transfer problem as a degenerate elliptic equation. Galerkin projection of this degenerate elliptic equation yields an equation for an operator that has a low-rank decomposition, which allows efficient application to objects which also exhibit a low-rank decomposition. To solve the projected equation, we consider a preconditioned Richardson iteration with rank control, which has been used in the context of high-dimensional elliptic equations in [2]. Due to the degeneracy of the elliptic equation, the construction of a preconditioner with a low-rank decomposition is challenging: we propose a suitable change of basis to highlight a Kronecker-sum structure of the new preconditioner, and we describe its application to low-rank object through exponential sums approximations and a particular summation procedure based on inexact evaluation of residuals.

The construction of the low-rank framework highlights the link between Hilbert space iterations and Linear Algebra, providing a solid motivation for the use of low-rank structure in the approximation of radiative transfer.

[1] - Egger, H. and Schlottbom, M., A mixed variational framework for the radiative transfer equation, Math. Models Methods Appl. Sci. 22 (2012)

[2] - Bachmayr, M. and Schneider, R., Iterative methods based on soft thresholding of hierarchical tensors, FoCM 17, 1037–1083 (2017)

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