Well-balanced high-order methods for systems of balance laws: recent results and applications

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This work summarizes some recent advances in the design of high-order well-balanced methods for systems of conservation laws by the authors and collaborators. The starting point is the family of high-order finite-volume methods proposed in [1] for hyperbolic 1d-systems of balance laws of the form

$$U_t + F(U)_x = S(U)H_x,$$

where H is a known possibly discontinuous function. The key ingredient in [1] was the reconstruction of fluctuations with respect to local equilibria. More precisely, once the numerical approximations at time t^n, U_i^n, have been obtained, the steps to compute the high-order reconstruction at the ith cell are the following: first, a stationary solution $U_i^*(x)$ of the system whose average at the ith-cell, $[x_{i-1/2}, x_{i+1/2}]$, is equal to U_i^n is obtained, i.e. U_i^* solves the problem

$$F(U_i^*)_x = S(U_i^*)H_x, \quad \frac{1}{\Delta x}\int_{x_{i-1/2}}^{x_{i+1/2}} U_i^*(x)\,dx = U_i^n.$$

Next, the fluctuations

$$V_j^n = U_j^n - \frac{1}{\Delta x}\int_{x_{j-1/2}}^{x_{j+1/2}} U_i^*(x)\,dx, \quad j \in \mathcal{S}_i$$

are computed, where $\mathcal{S}_i$ represents the stencil of the ith cell. Then, a standard high-order reconstruction operator (like MUSCL, ENO; WENO, CWENO, etc.) is applied to these fluctuations to obtain

$$Q_i(x) = Q_i(x; \{V_j^n\}_{j\in\mathcal{S}_i}).$$

Finally, the reconstruction operator is defined as follows:

$$P_i(x) = U_i^*(x) + Q_i(x).$$

This strategy has been since then extended to systems for which the analytic expression of the stationary solutions is not available neither in explict or implicit form. Moreover it has been extended to the design of implicit and assymptotic preserving well-balanced finite volume methods and it has been also succesfully extended to the design of well-balanced finite-difference, Discontinuous-Galerkin, or Taylor methods.

Well-balanced methods designed following this strategy have been applied to fluid models in different contexts: shallow water models, gas dynamic, relativistic fluid models, blood flow models, etc.

An overview of the different extensions and applications of this methodology will be presented, including a brief discussion about the extension to multidimensional problems.

REFERENCES:

[1] M.J. Castro, C. Parés. Well-balanced high-order finite volume methods for systems of balance laws. Journal of Scientific Computing 82 (2), 1-48, 2021.

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