Integration and function approximation on $\mathbb{R}^d$ using equispaced points and lattice points

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In this work, I will discuss integrating and approximating functions over $\mathbb{R}^d$ by equispaced points for $d = 1$ and lattice points for $d \geq 2$. In [1], together with D. Nuyens, we derived explicit conditions where lattice points can obtain error convergence of almost $n^{-\alpha}$ for integrating functions with smoothness $\alpha \in \mathbb{N}$ over the unbounded domain $\mathbb{R}^d$, where n is the number of quadrature points. When $d = 1$ and integration for α-smooth Gaussian Sobolev spaces is considered, in [2], together with Y. Kazashi and T. Goda, we proved that equispaced points achieve the optimal rate $n^{-\alpha}$ up to a logarithmic factor. In contrast, therein, the well known Gauss–Hermite quadrature was shown to achieve merely of the order $n^{-\alpha/2}$. Based on these results, I further consider the function approximation problem and possible use of lattice points on $\mathbb{R}^d$.

[1] D. Nuyens and Y. Suzuki *Scaled lattice rules for integration on $\mathbb{R}^d$ achieving higher-order convergence with error analysis in terms of orthogonal projections onto periodic spaces.* Mathematics of Computation, 92 (2023), pp. 307–347.

[2] Y. Kazashi, Y. Suzuki and T. Goda. *Sub-optimality of Gauss-Hermite quadrature and optimality of the trapezoidal rule for functions with finite smoothness.* Accepted for publication in SIAM Journal on Numerical Analysis.