A universal numerical integration by digital nets

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This talk concerns numerical integration over high-dimensional unit cubes, for which quasi-Monte Carlo (QMC) methods using (higher order) digital nets have been well studied. Our focus is to show that randomly chosen linearly scrambled nets or polynomial lattice point sets can achieve almost optimal rates of convergence (in the sense of worst-case error) for weighted Sobolev spaces with dominating mixed smoothness or even attain super-polynomial convergence for a class of infinitely differentiable functions with probability 1/2. By applying a median trick, we establish a universal QMC-based integration rule that does not require any information on smoothness or weights when constructing point sets, yet still achieves nearly optimal worst-case error convergence in various weighted function spaces with high probability. To support our theoretical claim, we present a series of numerical experiments that demonstrate the effectiveness of our approach.