L^2-approximation and numerical integration on Hermite spaces

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We consider spaces of functions of infinetely many variables that are defined with the help of all univariate Hermite polynomials, which (properly normalized) form an orthonormal basis of the space of all square-integrable functions over the real line endowed with the standard normal distribution. Those spaces belong to the class of reproducing kernel Hilbert spaces of increasing smoothness and their elements are defined on proper subsets of the sequence space (i.e., the space of all real-valued sequences). Their norms are induced by an underlying function space decomposition, namely the infinite-dimensional ANOVA decomposition. We discuss further properties of those spaces and present optimal results on L^2-approximation and on numerical integration.

The talk is based on the two recent papers

1) M. Gnewuch, M. Hefter, A. Hinrichs, K. Ritter, Countable tensor products of Hermite spaces and spaces of Gaussian kernels, Journal of Complexity 71 (2022), 101654. (arXiv:2110.05778v2)

2) M. Gnewuch, A. Hinrichs, K. Ritter, R. Rüssmann, Infinite-dimensional integration and L^2-approximation on Hermite spaces, preprint 2023, arXiv:2304.01754.