Weighted least-squares approximation in expected L^2 norm

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We investigate the problem of approximating a function u in L^2 with a linear space of functions of dimension n, using only evaluations of u at m chosen points. A first approach, based on weighted least-squares at i.i.d random points, provides a near-best approximation of u, but requires m of order $n\log(n)$. A reduction to a linear sample size, while preserving the quality of approximation can be obtained based on the solution to the Kadison-Singer problem. We improve on this result by using a randomized greedy strategy, which allows to reduce the oversampling ratio m/n and provides an algorithm of polynomial complexity.

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