The curse of dimensionality for the L_p-discrepancy with finite p

Erich Novak
FSU Jena, Germany
erich.novak@uni-jena.de

The L_p-discrepancy is closely related to the worst-case error of algorithms for numerical integration. Its inverse for dimension d and error threshold $\varepsilon \in (0,1)$ is the number of points in $[0,1]^d$ that is required such that the minimal normalized L_p-discrepancy is less or equal ε.

The inverse of L_2-discrepancy grows exponentially fast with the dimension d, i.e., we have the curse of dimensionality, whereas the inverse of L_∞-discrepancy depends linearly on d; both results are from 2001. The behavior of inverse of L_p-discrepancy for $p \notin \{2, \infty\}$ was open.

We show that the L_p-discrepancy suffers from the curse of dimensionality for all p of the form $p = 2\ell/(2\ell-1)$ with $\ell \in \mathbb{N}$. We prove lower bounds for the worst-case error of numerical integration in an anchored Sobolev space of once differentiable functions in each variable whose first derivative has finite L_q-norm, where q is an even positive integer with $1/p + 1/q = 1$.

Conjecture: The curse holds for all p with $1 < p < \infty$.

Joint work with Friedrich Pillichshammer (JKU Linz, Austria).