L^2-approximation and numerical integration on Gaussian Spaces

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We study L^2-approximation and integration on reproducing kernel Hilbert spaces $H(L_\sigma)$ of d variables, where $d \in \mathbb{N}$ or $d = \infty$. Here, L_σ is given as the tensor product of univariate Gaussian kernels, i.e., $L_\sigma(x, y) := \prod_{j=1}^{d} \exp(-\sigma_j^2 \cdot (x_j - y_j)^2)$. These spaces are closely related to Hermite spaces $H(K_\beta)$, where K_β is again of tensor product form, but based on univariate Hermite kernels, i.e., $K_\beta(x, y) := \prod_{j=1}^{d} \sum_{\nu=0}^{\infty} \beta_j^\nu \cdot h_\nu(x_j) \cdot h_\nu(y_j)$, where h_v is the Hermite polynomial of degree ν. More precisely, for each space $H(L_\sigma)$ there exists a corresponding space $H(K_\beta)$ and an isometric isomorphism Q between both spaces such that one function evaluation of Qf needs only one function evaluation of f and vice versa.

Via this correspondence, we are able to constructively transform any algorithm for L^2-approximation or integration on $H(K_\beta)$ into an algorithm for the same problem on $H(L_\sigma)$ and vice versa, preserving error and cost. In the case $d = \infty$, this allows us to investigate both problems on $H(L_\sigma)$ for the first time. In the case $d \in \mathbb{N}$, we are able to transfer some known results between the two function space settings.

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