On quadratures with optimal weights for spaces with bounded mixed derivatives

Michael Griebel
INS, University of Bonn, Germany
griebel@ins.uni-bonn.de

We discuss quadrature rules in d dimensions with optimal weights for given Quasi Monte Carlo point sets. Here we focus on spaces H^r_{mix} with bounded r-th mixed derivatives. It turns out that we obtain a convergence rate of the order $O(N^{-r}\log(N)^{\alpha(d,r)})$ for some α which depends on the dimension d and the smoothness r. The main order term N^{-r} is then substantially improved in contrast to conventional plain QMC, which achieves just a main order term of N^{-1}.

Moreover. we consider optimal weights for spaces H^s_{mix} with bounded s-th mixed derivatives and use these weights in a quadrature rule for functions from H^r_{mix}. There are now two situations: $s > r$ and $s < r$. We observe that we obtain an error of the order $O(N^{-\min(r,s)}\log(N)^{\beta(d,r,s)})$ for some β which depends on the dimension d and the smoothness r and s.

This way, if we do not know a-priorily the smoothness of our integrand class, it makes sense to choose optimal weights for some large r. Then, we always gain uniformly the best main error rate possible.

Joint work with Uta Seidler (INS, University of Bonn, Germany).