Spectral Picard-Vessiot theory and its applications

Sonia L. Rueda
Universidad Politécnica de Madrid, Spain
sonialuisa.rueda@upm.es

We investigate new techniques in differential and noncommutative algebra to develop the differential Galois theory for spectral problems, and apply these abstract theories to concrete differential spectral problems of integrable systems from Physics.

Given an ordinary differential operator L, with coefficients in a differential field, we study the spectral problem $LY = \lambda Y$, for a constant spectral parameter λ. We are interested in algebro-geometric linear differential operators, characterized by having nontrivial centralizers, and a spectral parameter governed by the famous spectral curve. In this situation we have coupled-spectral problems

$$LY = \lambda Y, \;\; AY = \mu Y,$$

for a non trivial ordinary differential operator A commuting with L. For instance, in the case of algebro-geometric Schrödinger operators $\partial^2 + u$, the potentials u are solutions of the KdV-hierarchy.

The recently defined spectral Picard-Vessiot field of a second order operator L provided a new approach to the factorization problem of ordinary differential operators in terms of parameters. The generalization to the case of prime-order operators appears naturally. Already the case of third-order operators, which are associated to the Boussineq integrable hierarchy, is an interesting and challenging problem, which has not been approached by differential Galoisian methods before.

In this talk I will present recent results and open questions related with this problems, in the framework of the project Algorithmic Differential Algebra and Integrability (ADAI).

Joint work with Maria-Angeles Zurro (Universidad Autónoma de Madrid), Juan J. Morales-Ruiz (Universidad Politécnica de Madrid) and Emma Previato (Boston University).