Galois groups for linear integrable systems of differential and difference equations over elliptic curves

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Let C be an algebraically closed field of characteristic zero, let E be an elliptic curve defined over C, and let K be the field of rational functions on E. Let δ be an invariant derivation on K (unique up to a constant multiple), and σ denote the automorphism on K induced by addition by a fixed non-torsion C-point of E under the elliptic group law. We consider a linear system

$$\delta(Y) = AY; \qquad \sigma(Y) = BY;$$

where $A \in \mathfrak{gl}_n(K)$ and $B \in \mathrm{GL}_n(K)$ satisfy the integrability condition

$$\delta(B) = \sigma(A)B - BA.$$

There are several (five!), a priori different and seemingly incomparable, Galois groups that one can attach to such a system. We explain why some of them must be abelian, and why (conjecturally) all of them must be solvable.

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