Integrable systems of the Boussinesq type and their formal diagonalisation

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We will deal with the symmetry approach to integrability for multi-component evolution systems $\mathbf{u} = \mathbf{\Phi}(\mathbf{u}, \mathbf{u}_1, \ldots, \mathbf{u}_n)$, or in components

$$u^i_t = \phi^i(\mathbf{u}, \mathbf{u}_1, \ldots, \mathbf{u}_n), \quad i = 1, \ldots, m$$

where $\mathbf{u} = (u^1, \ldots, u^m)$ and $\mathbf{u}_i = \partial \mathbf{u}/\partial x^i$. Formal recursion $\mathbf{R}$ and symplectic $\mathbf{S}$ operators are matrix pseudo-differential series satisfying the equations

$$\mathbf{R}\_t = [\mathbf{\Phi}_*, \mathbf{R}],$$
$$\mathbf{S}\_t + \mathbf{S}\,\mathbf{\Phi}\_* + \mathbf{\Phi}\_*^+ \ \mathbf{S} = 0.$$

where $\mathbf{\Phi}_*$ denotes the Fréchet derivative of $\mathbf{\Phi}$. Integrability amounts to the existence of such operators for a given system.

Non-degenerate systems are those having its separant matrix $\mathbf{\Sigma}$ (with entries $\sigma_{ij} = \partial \phi^i / \partial u^j_n$) invertible and without multiple eigenvalues at a generic point. It is well known that starting from the transformation that diagonalises the separant, a transformation that formally diagonalises the whole system can be built, helping to solve the equations for recursion and symplectic operators, i.e. to establish integrability.

We will explore in this talk the possibility of diagonalising degenerate systems, i.e. with non diagonalisable separants. As an illustration, we will show some classifications of integrable systems of the form

$$u_t = v,$$
$$v_t = u_4 + f(u, u_1, \ldots, u_3, v, v_1).$$

The computational component of this work consist in developing and implementing the formal calculus of pseudo-differential expressions in the jet space.

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