Algebraic consequences of the fundamental theorem of calculus in differential rings

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In this talk, we discuss the fundamental theorem of calculus and its consequences from an algebraic point of view [1]. In particular, for functions with singularities, this leads to a generalized notion of evaluation. We present properties of such integro-differential rings and discuss several examples. We outline the construction of the corresponding integro-differential operators and provide normal forms using rewrite rules. These rewrite rules are then used to derive several identities and properties in a purely algebraic manner, generalizing well-known results from analysis. In identities such as shuffle relations for nested integrals and the Taylor formula, additional terms are obtained to account for singularities. Another focus lies on treating the basics of linear ordinary differential equations (ODEs) within the framework of integro-differential operators. These operators can have matrix coefficients, enabling the treatment of systems of arbitrary size in a unified manner.

[1] Clemens G. Raab and Georg Regensburger. The fundamental theorem of calculus in differential rings. arXiv:2301.13134 [math.RA] (2023)

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