Darboux Transformations for Orthogonal Differential Systems and Differential Galois Theory

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Darboux developed an ingenious algebraic mechanism to construct infinite chains of "integrable" second order differential equations as well as their solutions. After a surprisingly long time, Darboux's results were rediscovered and applied in many frameworks, for instance in quantum mechanics (where they provide useful tools for Supersymmetric Quantum Mechanics), in soliton theory, Lax pairs and many other fields involving hierarchies of equations. In this work, we propose a method which allows us to generalize the Darboux transformations algorithmically for tensor product constructions on linear differential equations or systems. We obtain explicit Darboux transformations for third order orthogonal systems ($\mathfrak{so}(3, C_K)$ systems) as well as a framework to extend Darboux transformations to any symmetric power of $\mathrm{SL}(2, \mathbb{C})$-systems.

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