Computation of confluent hypergeometric functions

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Confluent hypergeometric functions appear in many applications in applied mathematics, physics and engineering. Despite their importance, few algorithms are available for computing any of the standard solutions of Kummer's equation in the case of real or complex parameters. In this talk, we present recent advances in the computation of the Kummer function $U(a, b, x)$ for real values of the parameters [1] and in the evaluation of a particular case of the Kummer function, the parabolic cylinder function $U(a, z)$ for complex arguments [2].

On the other hand, confluent hypergeometric functions play a key role in the asymptotic analysis of Fermi-Dirac integrals. The evaluation of these integrals and their derivatives is crucial for computations in physical problems dominated by the degenerate fermions. Therefore, the Fermi-Dirac integrals are widely used in stellar astrophysics or plasma physics. In this lecture we will show that the use of asymptotic expansions allow to calculate these integrals efficiently and with high accuracy for a large range of parameters.

References

[1] A. Gil, D. Ruiz-Antolín, J. Segura, N.M. Temme. Efficient and accurate computation of confluent hypergeometric function $U(a, b, x)$. Submitted.

[2] T.M. Dunster, A. Gil, J. Segura. Computation of parabolic cylinder functions having complex argument. Submitted.

[3] A. Gil, J. Segura, N.M. Temme. Complete asymptotic expansions for the relativistic Fermi-Dirac integral. Appl. Math. Comput. 412 (2022) 126618.

[4] A. Gil, A. Odrzywolek, J. Segura, N.M. Temme. Evaluation of the relativistic Fermi-Dirac integral and its derivatives for moderate/large values of the parameters. Comput. Phys. Commun. 283 (2023) 108563.

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