Sobolev orthogonal polynomials for solving the Schrödinger equation with potentials $V(x) = x^{2k},\ k \geqslant 2$.

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The variational formulation of a boundary value problem for the harmonic oscillator with potentials $V(x) = x^{2k}$, $k \geqslant 2$, given by

$$\begin{aligned} -\&u'' + \lambda\, x^{2k}\, u &= f(x), \\ \&u(-1) = u(1) &= 0, \end{aligned}$$

for $\lambda > 0$, provides a Sobolev inner product, that is, an inner product involving derivatives that can be deeply studied, and used to solve the BVP.

In this work we study the associated family of Sobolev orthogonal polynomials, including a recursive way to compute them as well as the outer relative asymptotics of this polynomials and classical Legendre polynomials.

The analysis of the Fourier-Sobolev coefficients and some numerical experiments complete this talk.

Joint work with Lidia Fernández (Universidad de Granada, Spain), Francisco Marcellán (Universidad Carlos III, Spain) and Miguel A. Piñar (Universidad de Granada, Spain).