Mehler-Heine asymptotics for q-hypergeometric polynomials

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The basic q-hypergeometric function ${}_r\phi_s$ is defined by the series

$${}_r\phi_s\left(\begin{array}{c} a_1,\dots,a_r \\ b_1,\dots,b_s \end{array};q,z\right)=\sum_{k=0}^{\infty}\frac{(a_1;q)_k\cdots(a_r;q)_k}{(b_1;q)_k\cdots(b_s;q)_k}\left((-1)^k q^{\binom{k}{2}}\right)^{1+s-r}\frac{z^k}{(q;q)_k},$$

where $0<q<1$ and $(a_j;q)_k$ and $(b_j;q)_k$ denote the q-analogues of the Pochhammer symbol.

When one of the parameters a_j is equal to q^{-n} the basic q-hypergeometric function is a polynomial of degree at most n in the variable z. Our objective is to obtain a type of local asymptotics, known as Mehler-Heine asymptotics, for q-hypergeometric polynomials.

Concretely, by scaling adequately these polynomials we get a limit relation between them and a q-analogue of the Bessel function of the first kind. Originally, this type of local asymptotics was introduced for Legendre orthogonal polynomials (OP) by the German mathematicians H. E. Heine and G. F. Mehler in the 19th century. Later, it was extended to the families of classical OP (Jacobi, Laguerre, Hermite), and more recently, these formulae were obtained for other families as generalized Freud OP, multiple OP or Sobolev OP, among others.

These formulae have a nice consequence about the scaled zeros of the polynomials, i.e. using the well-known Hurwitz's theorem we can establish a limit relation between these scaled zeros and the ones of a Bessel function of the first kind. In this way, we deduce a similar relation in the context of the q-analysis and we will illustrate this with numerical examples. The results have recently been published in [1].

[1] J.F. Mañas–Mañas, J.J. Moreno–Balcázar, Asymptotics for some q-hypergeometric polynomials, Results Math. 77(4), Art. 146 (2022).

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