A least squares analog to the Nuttall-Pommerenke theorem.

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In various contexts involving identification and design, the following least-squares substitute to multipoint Padé approximation became quite popular in recent years under the name of "vector fitting": given a holomorphic function f and a set of points $z_1, \cdots, z_N$ in the complex plane, to find a rational function p_m/q_n of type (m, n) minimizing the criterion $\sum_{j=1}^{N} |q(z_j)f(z_j) - p(z_j)|^2$. This type of approximation involves non-classical orthogonality, and its behaviour is still fairly open.

We analyze here the classical Padé analog where one minimizes the l^2-norm of the first n+m+1 terms of the Taylor expansion of the linearized error at a point z_0. In particular, we prove that convergence in capacity prevails when f is analytic on the complex plane minus a polar set; i.e., a set of logarithmic capacity zero, provided that $N \leq C(n+m)$. This least-square version of the Nutall-Pommerenke theorem also sheds light on the multipoint case.

Joint work with Paul Asensio.