Approximate Calculation of Sums based on a Gaussian type quadrature

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The best approximation polynomial with respect to a table with large amount of data by the method of least squares can be given, in a stable way, in an orthogonal basis. To find this polynomial is necessary to calculate numerical sums with large amount of terms. We proposed an algorithmic approach for approximate calculation of sums of the form $\sum_{j=1}^{N} f(j)$ based on a Gaussian type quadrature formula for sums. This allows the calculation of sums with very large number of terms N to be reduced to sums with a much smaller number of summands n. For use the Gaussian type quadrature we need the n nodes that are the zeros of Gram polynomials, also known as Discrete Chebyshev polynomials. In [1] we obtained precise lower and upper bounds for these zeros. We proved in [2] that the Weierstrass–Dochev–Durand–Kerner iterative numerical method, with explicitly given initial conditions, converges to the nodes of the quadrature formula. Several methods for computing the nodes of the discrete analogue of the Gaussian quadrature formula are compared. Since, for practical purposes, any approximation of a sum should use only the values of the summands, we implement a simple but efficient procedure to additionally approximate the evaluations at the nodes by local natural splines. Explicit numerical examples are provided. Moreover, the error in different spaces of functions was analyzed.

Bibliography [1] Area, I., Dimitrov, D. K., Godoy E., Paschoa, V. G. . Approximate Calculation of Sums I: Bounds for the Zeros of Gram Polynomials. *SIAM Journal on Numerical Analysis*, v. 52, p. 1867-1886, 2014.

[2] Area, I., Dimitrov, D. K., Godoy E., Paschoa, V. G. Approximate Calculation of Sums II: Gaussian Type Quadrature. *SIAM Journal on Numerical Analysis*, v. 54, p. 2210-2227, 2016.

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