On a new class of classical 2-orthogonal polynomials

Khalfa Douak
Sorbonne Université - Campus Pierre et Marie Curie, France
khalfa.douak@gmail.com

In this talk, I will present recent results on 2-orthogonal polynomials obtained in two joint papers with Pascal Maroni [1], [2]. This constitutes a few contribution in the area of the d-orthogonal polynomials. Thanks to the Hahn property defining the classical character, we first give a new class of classical 2-orthogonal polynomials for which we determine the explicit expressions of the recurrence coefficients with some other properties. Six special cases of those polynomials have been pointed out. The aim of the second part is to seek integral representations for the pair of linear functionals with respect to which such polynomials are 2-orthogonal. We start with the matrix differential equation satisfied by the vector of those functionals which, in turn, provides differential equations satisfied by the respective (sought) weight functions. Depending on the case, we obtain that these weights are defined in terms of various special functions with supports on the real line or on positive real line. In order for certain integral representations to exist, addition of Dirac masses at the origin is necessary.

References:

[1] K. Douak , P. Maroni. On a new class of 2-orthogonal polynomials, I: The recurrence relations and some properties. Integral Trans. Spec. Funct., 32(2) (2021) 134–153.

[2] K. Douak , P. Maroni. On a new class of 2-orthogonal polynomials, II: The integral representations, arXiv:2212.11949v2 (2023) 1–30.

Joint work with Pascal Maroni.