Resonances as a computational tool

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A large toolbox of numerical schemes for dispersive equations has been established, based on different discretization techniques such as discretizing the variation-of-constants formula (e.g., exponential integrators) or splitting the full equation into a series of simpler subproblems (e.g., splitting methods). In many situations these classical schemes allow a precise and efficient approximation. This, however, drastically changes whenever non-smooth phenomena enter the scene such as for problems at low regularity and high oscillations. Classical schemes fail to capture the oscillatory nature of the solution, and this may lead to severe instabilities and loss of convergence. In this talk I present a new class of resonance based schemes. The key idea in the construction of the new schemes is to tackle and deeply embed the underlying nonlinear structure of resonances into the numerical discretization. As in the continuous case, these terms are central to structure preservation and offer the new schemes strong geometric properties at low regularity.

I will present the key idea behind resonances as a computational tool, their high order counterpart (via tree series inspired by singular SPDEs), their error estimates in low regularity spaces (via discrete Bourgain spaces) and numerical experiments on nonlinear dispersive quantization effects. I also want to address the fundamental question of resonance based schemes and structure preservation, i.e., central symmetries and even more so symplecticity.