Towards a Fluid computer

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In 1991, Moore [4] raised a question about whether hydrodynamics is capable of performing computations. Similarly, in 2016, Tao [6] asked whether a mechanical system, including a fluid flow, can simulate a universal Turing machine. Etnyre and Ghrist showed in [3] that contact geometry and fluid dynamics are related through a mirror that reflects Reeb vector fields as Beltrami vector fields, allowing us to answer these questions.

In this talk, we will present the construction in [1] of a "Fluid computer" in dimension 3 that uses this "mirror" combining techniques developed by Alan Turing with symbolic dynamics and modern Geometry (contact geometry). A completely different construction for the Euclidean metric is given in [2]. These constructions reveal the existence of undecidable fluid paths. Tao's question was motivated by a research program to address the Navier–Stokes existence and smoothness problem ([5] and [6]). Could such a Fluid computer be used to address this Millennium prize problem?

Time permitting, we will end up the talk with some speculative ideas of a Fluid computer construction à la Feynman.

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[2] R. Cardona, E. Miranda, and D. Peralta-Salas, Computability and Beltrami fields in Euclidean space. J. Math. Pures Appl. (9) 169 (2023), 50–81.

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[4] C. Moore, Generalized shifts: Unpredictability and undecidability in dynamical systems. Nonlinearity 4, 199 (1991).

[5] T. Tao, Searching for singularities in the Navier–Stokes equations. Nat. Rev. Phys. 1, 418–419 (2019).

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