Multiple orthogonal polynomials and special functions

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Multiple orthogonal polynomials are polynomials in one variable that satisfy orthogonality conditions with respect to r measures. They appear naturally in Hermite-Padé approximation to r functions. The case $r = 1$ corresponds to the usual orthogonal polynomials. Several systems of multiple orthogonal polynomials have been constructed using classical weight functions (multiple Hermite, multiple Laguerre, multiple Jacobi polynomials). In this talk I will use weight functions given by special functions satisfying a differential equation. The r weights then appear by writing the differential equation as a system of first order equations, which then generalizes the Pearson equation for classical orthogonal polynomials. The weights are in terms of modified Bessel functions $K_\nu(2\sqrt{x})$, modified Bessel functions $I_\nu(2\sqrt{x})$, hypergeometric functions, confluent hypergeometric functions, and the exponential integral. We give some applications where these multiple orthogonal polynomials appear, such as eigenvalues and singular values of products of random matrices, non-intersecting Brownian motions, and rational approximations for real numbers.