Least-squares solvers for PDEs

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Least squares solvers regain popularity because of their suitability for approximation with neural networks. After recalling the least-squares approach to numerically approximate the solution of a PDE, we will present techniques to obtain quasi-optimal approximations also in the case that the residual is measured in a norm that cannot be evaluated, as a dual norm or fractional Sobolev norm. The latter norms typically arise when the PDE is appended with inhomogeneous boundary conditions. Dual norms are unavoidable with so-called ultra-weak formulations of PDEs. By employing with such formulations the so-called optimal test norm, one is able to approximate the best approximation from the trial space within any tolerance.

As an application of the latter approach, we present details about an ultra-weak first order system system discretization of the Helmholtz equation that is pollution-free.